

21004973

MATHEMATICS

Maximum Marks: 80

Time Allotted: Three Hours

Reading Time: Additional Fifteen Minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The Question Paper has **11 printed pages and one blank page**.
4. The Question Paper is divided into **three** sections and has **22 questions** in all.
5. **Section A** is compulsory and has **fourteen** questions.
6. You are required to attempt **all** questions either from **Section B or Section C**.
7. **Section B** and **Section C** have **four** questions each.
8. Internal choices have been provided in **two** questions of **2 marks**, **two** questions of **4 marks** and **two** questions of **6 marks** in **Section A**.
9. Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks** each in **Section B and Section C**.
10. While attempting **Multiple Choice Questions** in **Section A, B and C**, you are required to **write only ONE** option as the answer.
11. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
12. Mathematical tables and graph papers are provided.
13. The intended marks for questions or parts of questions are given in the brackets [].

Instruction to Supervising Examiner

1. Kindly read **aloud** the instructions given above to all the candidates present in the examination hall.

SECTION A – 65 MARKS

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$, then the values of x are: [1]

(a) $0, \frac{-3}{2}$

(b) $0, \frac{3}{2}$

(c) $0, \frac{2}{3}$

(d) $0, \frac{-2}{3}$

(ii) The equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1} \left(\frac{1}{2} \right)$ has: [1]

(a) No solution

(b) Two solutions

(c) Unique solution

(d) Infinitely many solutions

(iii) If $A = \{1,2,3\}$ and the relation $R = \{(1,1), (2,2), (3,3), (2,3)\}$, then [1]

Assertion: R is not an equivalence relation.

Reason: R is a reflexive relation.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(iv) If A and B are matrices of same order, then $AB^T - BA^T$ is a: [1]

(a) Null matrix

(b) Unit matrix

(c) Symmetric matrix

(d) Skew symmetric matrix

- (v) Consider the matrix A of order 3×3 with $|A| = 5$. If any two columns are interchanged in A , then $|A|$ is: [1]

- (a) -5
- (b) $(-5)^3$
- (c) 5
- (d) 5^3

- (vi) **Statement 1:** The sum, difference and product of two continuous functions are also continuous. [1]

Statement 2: The function $f(x)$ is differentiable on (a, b) implies that the function $f(x)$ is differentiable on $[a, b]$.

Which one of the following is correct?

- (a) Statement 1 is true and Statement 2 is false.
- (b) Statement 2 is true and Statement 1 is false.
- (c) Both the statements are true.
- (d) Both the statements are false.

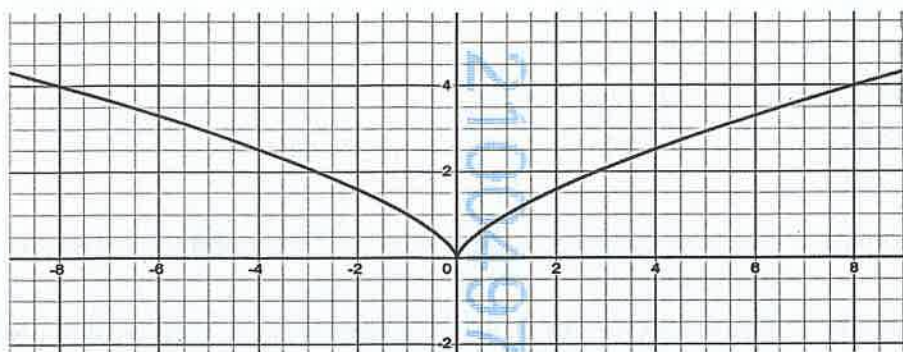
- (vii) The value of $\int_0^1 |x + 1| dx$ is: [1]

- (a) 0
- (b) $\frac{3}{2}$
- (c) $\frac{1}{2}$
- (d) $\frac{-3}{2}$

- (viii) A die is thrown three times. If the first throw is a four, then the probability of getting 15 as a sum is: [1]

- (a) $\frac{1}{18}$
- (b) $\frac{1}{108}$
- (c) $\frac{1}{72}$
- (d) $\frac{1}{36}$

- (ix) Consider the graph given below of the function $y = x^{\frac{2}{3}}$ and answer the question that follows: [1]



Statement 1: The function is continuous at $x = 0$.

Statement 2: The function is differentiable at $x = 0$.

Which one of the following is correct?

- (a) Statement 1 is true and Statement 2 is false.
 - (b) Statement 2 is true and Statement 1 is false.
 - (c) Both the statements are true.
 - (d) Both the statements are false.
- (x) The system of linear equations $x + y + z = 6$, $x + \lambda y + 2z = 7$ and $3x + y + z = 12$ has a unique solution if: [1]

- (a) $\lambda = 2$
- (b) $\lambda \neq 2$
- (c) $\lambda = -2$
- (d) $\lambda \neq -2$

- (xi) **Assertion:** If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, then $x + y + xy = 1$ [1]

Reason: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$, $xy < 1$

Which one of the following is correct?

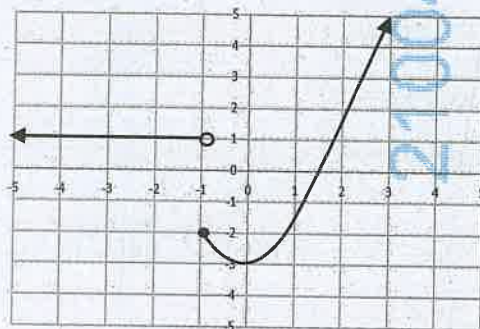
- (a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
 - (b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
 - (c) Assertion is true and Reason is false.
 - (d) Assertion is false and Reason is true.
- (xii) Evaluate: $\int \frac{10^x \log 10 + 10x^9}{10^x + x^{10}} dx$ [1]

- (xiii) Find the general solution for the following differential equation:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

- (xiv) Differentiate $y = x^x$ with respect to x .

- (xv) Find the range of the function shown in the graph below:



Question 2

- (i) Prove that the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$, defined by $f(x) = x - 5$ is a bijective function.

OR

- (ii) For what values of ' x ' is the function $f(x) = \frac{4x+3}{6x-4}$ invertible? Also, find the inverse function.

Question 3

Using determinants, find the value of x if the points $(x, -2)$, $(5, 2)$ and $(8, 8)$ are collinear.

Question 4

- (i) A water treatment plant uses a large cylindrical storage tank to collect treated water before distribution. The tank has a diameter of 20 meter. During the filling process, water flows into the tank at a constant rate of 314 cubic meter per hour. To monitor the process, an engineer records the rate of change of the depth of water in the tank. Find the value that the engineer recorded.

OR

- (ii) Find the equation of the tangent to the curve $x^2 + xy - 3 = 0$ at $(1, 2)$.

Question 5

Prove that $y = \frac{1}{2}(-8x^2 + 4x + 5)$ is an increasing function in $(-\infty, \frac{1}{4}]$.

Question 6

[2]

If A and B are two independent events such that $P(\overline{A \cup B}) = \frac{1}{3}$ and $P(\overline{A}) = \frac{2}{3}$ calculate $P(B)$.

Question 7

(i) If $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ then,

(a) Prove that $6x^2 + 5x - 1 = 0$. [2]

(b) Hence, solve the equation $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$. [2]

OR

(ii) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then find the value of $x^2 + y^2 + z^2 + 2xyz$. [4]

Question 8

(i) (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, then find A^{-1} . [3]

(b) Using A^{-1} , solve the system of linear equation given below: [1]

$$x - 2y = -5, 2x - y - z = -3, -2y + z = -4$$

OR

(ii) Using properties of determinants, solve for x: $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = 0$. [4]

Question 9

[4]

If $y = x + \tan x$, show that $\cos^2 x \frac{d^2 y}{dx^2} - 2y + 2x = 0$

Question 10

[4]

A football club purchased footballs manufactured at three different factories in Chennai, Mumbai and Delhi.

Given below is the data of the footballs manufactured at these factories during the same period:

- Chennai factory produced twice as many footballs as the Mumbai factory.
- The factory in Delhi manufactured the same number of footballs as that of the Mumbai factory.

Past records show that:

- 0.2% of the footballs produced by factories at Chennai and Mumbai were defective.
- 0.4% of the footballs manufactured at Delhi had defects.

A quality control team chose a football and found it to be defective.

What is the probability that it was produced in Mumbai factory?

Question 11

A manufacturing company produces airtight cylindrical containers for storing sensitive chemicals. For safety, each container is placed inside a spherical protective shell of radius 'r' cm. The cylindrical container is so designed that it fits completely inside the sphere, with its axis passing through the centre of the sphere. The company wants to utilise the space better and wants the cylindrical container to occupy the maximum possible volume inside the spherical shell.

To achieve this, the design team applies the concept of Maxima and Minima to determine the optimal dimensions of the cylindrical container.

Answer the following questions based on the above situation:

- (i) Obtain a relation between the height 'h' and the radius 'x' of the cylinder. [1]
- (ii) Write the expression for the volume 'V' of the cylindrical container in terms of 'x' and 'r'. [1]
- (iii) Find the value of 'x' for which the volume of the cylinder is maximum. [3]
- (iv) Hence, find the maximum volume of the cylindrical container. [1]

Question 12

[6]

- (i) Evaluate: $\int \frac{dx}{x^3+x^2+x+1}$

OR

- (ii) Using properties of definite integral, prove that: $\int_0^\pi \frac{x}{1+\sin x} dx = \pi$

Question 13

[6]

- (i) Find the particular solution of the following differential equation:
 $x^2 dy - (2xy + y^2) dx = 0$ given that $y = 1$ when $x = 1$.

OR

- (ii) Solve the differential equation $\frac{dy}{dx} + y \tan x = \sin 2x$, when $x = 0$ and $y = 1$.

Question 14

A school conducted an inter house quiz competition. The number of students who participated in the competition was 15. The scores of the children in the quiz competition were as follows:

14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19, 20

Let X be the random variable which represents the score of the students.

- (i) Form a frequency table for the scores. [1]
- (ii) Prepare the probability distribution table of ' X '. Verify if it is a valid probability distribution or not. [2]
- (iii) Find the expected score from the probability distribution table. [2]
- (iv) How many more marks are additionally required to attain the expected score of 20? [1]

SECTION B - 15 MARKS

Question 15

In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed.

- (i) For any three vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$, [1]

Statement 1: $\vec{a} \cdot (\vec{b} \times \vec{c})$ is a scalar.

Statement 2: $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{c} \times \vec{b})$

Which one of the following is correct?

- (a) Statement 1 is true and Statement 2 is false.
- (b) Statement 2 is true and Statement 1 is false.
- (c) Both the statements are true.
- (d) Both the statements are false.

- (ii) A unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$ is: [1]

- (a) $\frac{6\hat{i} - \hat{k}}{\sqrt{37}}$
- (b) $\frac{6\hat{i} + 6\hat{j} - \hat{k}}{\sqrt{73}}$
- (c) $\frac{6\hat{i} + \hat{k}}{\sqrt{37}}$
- (d) $\frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$

(iii) For vector $\vec{c}$, $\vec{c} \cdot \vec{i} = \vec{c} \cdot \vec{j} = \vec{c} \cdot \vec{k}$ and $|\vec{c}| = 300$. Then $\vec{c}$ is equal to: [1]

(a) $10\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

(b) $\frac{100}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

(c) $\frac{10}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

(d) $100\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

(iv) Find the equation of the plane parallel to ZOX plane with Y intercept 3. [1]

(v) Find the angle between the lines $\frac{x+3}{3} = \frac{y-1}{2} = \frac{z-2}{4}$ and the plane $2x + y - 3z + 4 = 0$. [1]

Question 16 [2]

(i) A school's media team identifies three points A, B and C on a large bulletin board to position three charts. The position vectors of A, B and C are:

$$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k} \text{ and } \vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k} \text{ respectively.}$$

They are placed in such a way that a triangle is formed. Find the area of the triangle ABC.

OR

(ii) Three forces act simultaneously on a space satellite in such a way that the satellite remains in equilibrium. The forces are represented by three vectors $\vec{p}$, $\vec{q}$ and $\vec{r}$, such that $\vec{p} + \vec{q} + \vec{r} = 0$, $|\vec{p}| = 3$, $|\vec{q}| = 5$ and $|\vec{r}| = 7$.

Find the angle between the forces $\vec{p}$ and $\vec{q}$.

Question 17 [4]

(i) Find the equation of the plane passing through the points $(2, -3, 1)$ and $(-1, 1, -7)$ and perpendicular to the plane $x - 2y + 5z + 1 = 0$.

OR

(ii) Find the foot of perpendicular from the point $P(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

Question 18 [4]

Find the area of the region bounded by the curves $y^2 = x$ and $y^2 = 4 - 3x$.

SECTION C – 15 MARKS

Question 19

In subparts (i) and (ii) choose the correct options and in subpart (iii) to (v), answer the questions.

- (i) **Statement 1:** Profit is maximum when marginal revenue equals marginal cost. [1]
Statement 2: At maximum profit, the slope of the profit function is zero.

Which one of the following is correct?

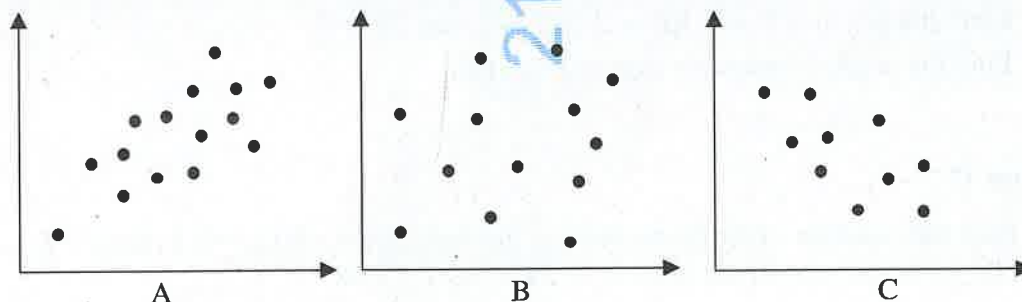
- (a) Statement 1 is true and Statement 2 is false.
 (b) Statement 2 is true and Statement 1 is false.
 (c) Both the statements are true.
 (d) Both the statements are false.
- (ii) If $\bar{x} = 28$, $\bar{y} = 32$, $b_{yx} = 1 \cdot 8$ and $b_{xy} = 0 \cdot 5$, then the regression equation of y on x is: [1]

- (a) $y = 1 \cdot 8x - 18 \cdot 4$
 (b) $y = 22x + 10$
 (c) $y = 1 \cdot 8x + 18 \cdot 4$
 (d) $y = 22x + 18$

- (iii) The total cost function and the total revenue function of a commodity are given by $C(x) = x + 40$ and $R(x) = ax$ respectively. Find the value of 'a' if the breakeven point is 5 unit. [1]

- (iv) A company produces a product whose cost function is $C(x) = x^3 - 27x + 2000$. Find the number of products when the marginal cost is 0. [1]

- (v) [1]



Justify which scatter diagram given above represents a regression equation with positive slope.

Question 20

- (i) The cost of manufacturing x units of a commodity is $5x^2 + 30x + 5$. Find the range of output where Average Cost is increasing. [2]

OR

- (ii) The Revenue function for x units of a commodity is $R(x) = x^3 + 3x^2 - 7x + 16$. Calculate the Marginal Revenue at $x = 5$.

Question 21

A school runs two training modules for sports competition. The data for a sample of 6 batches is given below.

X: Average number of hours of extra training per batch

Y: Average improvement in scores per batch

$$\sum X = 30, \quad \sum Y = 42, \quad \sum XY = 199, \quad \sum X^2 = 184, \quad \sum Y^2 = 318$$

Using the above data, find:

- (i) The regression coefficient of Y on X and X on Y. [2]
- (ii) Calculate the average improvement in scores if the average training hours are 10. [2]

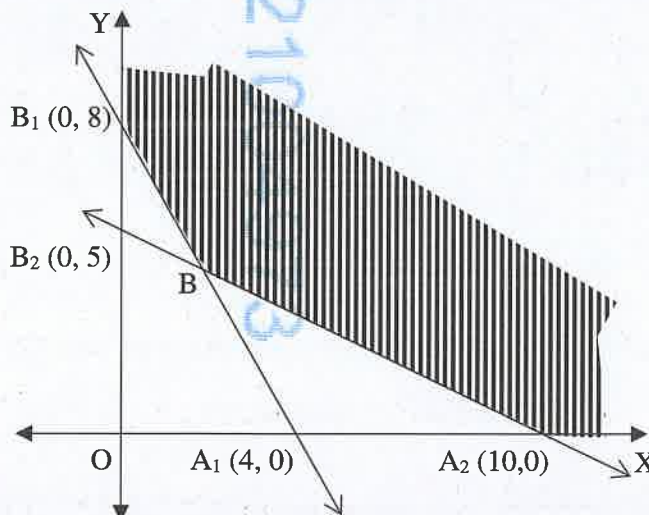
Question 22

- (i) An ice-cream factory makes ice creams of two flavours: chocolate and mango. [4]
 - Each chocolate ice cream needs 5 minutes of mixing and 10 minutes of freezing.
 - Each mango ice cream needs 8 minutes of mixing and 8 minutes of freezing.
 - There are 3 hours 20 minutes available for mixing and 240 minutes available for freezing.
 - The profit from the sale of each chocolate ice cream is ₹50 and from each mango ice cream is ₹60.

Using Linear Programming Problem, find the number of chocolate and mango ice creams that the ice cream factory should make to maximise profit.

OR

- (ii) The feasible region for a Linear Programming Problem is shown in the following graph.



Based on the given graph, answer the following questions:

- (a) Write the constraints for the Linear Programming Problem. [2]
- (b) Find the coordinates for point B. [1]
- (c) Find the least value of the objective function: [1]

$$Z = x + y$$

SEAL