MATHEMATICS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

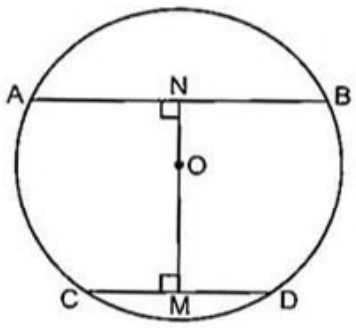
Question 1.

(a) *If b is the mean proportion between a and c, show that :* **[3]**

$$\frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4} = \frac{a^2}{c^2}$$

(b) *Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places.* **[4]**

(c) *AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords.*** **[3]**



Answer 1.

(a) Given, b is mean proportional between a and c.

$$\therefore \quad \frac{a}{b} = \frac{b}{c} = k \text{ (say)}$$

$$\Rightarrow \quad b = kc;\ a = kb = k^2c$$

$$\therefore \quad \text{L.H.S.} = \frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$$

$$= \frac{(k^2c)^4 + (k^2c)^2 \cdot (kc)^2 + (kc)^4}{(kc)^4 + (kc)^2 c^2 + c^4}$$

$$= \frac{k^8c^4 + k^6c^4 + k^4c^4}{k^4c^4 + k^2c^4 + c^4}$$

$$= \frac{k^4c^4 (k^4 + k^2 + 1)}{c^4 (k^4 + k^2 + 1)} = k^4$$

$$\text{R.H.S.} = \frac{a^2}{c^2} = \frac{(k^2c)^2}{c^2} = \frac{k^4c^2}{c^2} = k^4$$

$$\therefore \quad \text{L.H.S.} = \text{R.H.S.}$$ **Hence Proved.**

(b) Given equation is, $4x^2 - 5x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$,

We have, $a = 4, b = -5, c = -3$

** **Answer is not given due to change in the present syllabus.**

We know, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$

$= \frac{5 \pm \sqrt{25 + 48}}{8} = \frac{5 \pm \sqrt{73}}{8} = \frac{5 \pm 8{\cdot}544}{8}$

$= \frac{5 + 8{\cdot}544}{8}$ or $\frac{5 - 8{\cdot}544}{8}$

$= \frac{13{\cdot}544}{8}$ or $\frac{-3{\cdot}544}{8}$

$= 1{\cdot}693$ or $-0{\cdot}443$

$= 1{\cdot}69$ or $-0{\cdot}44$ (correct to 2 decimal places) **Ans.**

Question 2.

(a) *Evaluate without using trigonometric tables,*** **[3]**

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

(b) *If* $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ *and* $B \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ *and* $A^2 - 5B^2 = 5C$. *Find matrix C where C is a 2 by 2 matrix.* **[4]**

(c) *Jaya borrowed ₹ 50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹ 33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt.*** **[3]**

Answer 2.

(b) Given, $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$

$A^2 - 5B^2 = 5C$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} = 5C$

$\Rightarrow 5C = \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix} - 5\begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix}$

$\Rightarrow 5C = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 5C = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$\Rightarrow 5C = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$

$\Rightarrow C = \frac{1}{5}\begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ **Ans.**

** **Answer is not given due to change in the present syllabus.**

Question 3.

(a) *The catalogue price of a computer set is ₹ 42000. The shopkeeper gives a discount of 10% on the listed price. He further gives an off-season discount of 5% on the discounted price. However, sales tax at 8% is charged on the remaining price after the two successive discounts. Find :*** [3]

(i) the amount of sales tax a customer has to pay

(ii) the total price to be paid by the customer for the computer set.

(b) *P(1, – 2) is a point on the line segment A(3, – 6) and B(x, y) such that AP : PB is equal to 2 : 3. Find the coordinates of B.* [4]

(c) *The marks of 10 students of a class in an examination arranged in ascending order is as follows :* [3]

13, 35, 43, 46, x, x + 4, 55, 61, 71, 80

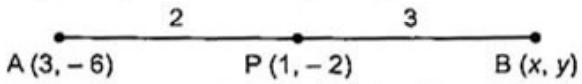
If the median marks is 48, find the value of x. Hence find the mode of the given data.

Answer 3.

(b) Given equation is P (1, – 2), A (3, – 6), B (x, y), AP : PB = 2 : 3

By section formula,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2},\ y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$



$$\Rightarrow \quad 1 = \frac{2 \times x + 3 \times 3}{2 + 3},\ -2 = \frac{2 \times y + 3 \times (-6)}{2 + 3}$$

$$\Rightarrow \quad 5 = 2x + 9, \quad -10 = 2y - 18$$

$$\Rightarrow \quad 2x = 5 - 9, \quad 2y = -10 + 18$$

$$\Rightarrow \quad x = \frac{-4}{2}, \quad y = \frac{8}{2}$$

$$\Rightarrow \quad x = -2, y = 4$$

The coordinates of B = (– 2, 4) **Ans.**

(c) Given marks are 13, 35, 43, 46, x, $x + 4$, 55, 61, 71, 80

Median = 48

$\because$ $n = 10$ (even)

$$\therefore \quad \text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{observation}\right]$$

$$= \frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2}$$

$$= \frac{x + x + 4}{2}$$

$$\therefore \quad 48 = \frac{2x + 4}{2}$$

$$\Rightarrow \quad 48 = \frac{2(x + 2)}{2}$$

$$\Rightarrow \quad x = 48 - 2 = 46$$

** **Answer is not given due to change in the present syllabus.**

$\therefore \quad x + 4 = 46 + 4 = 50$

$\therefore$ The numbers are : 13, 35, 43, 46, 46, 50, 55, 61, 71, 80.

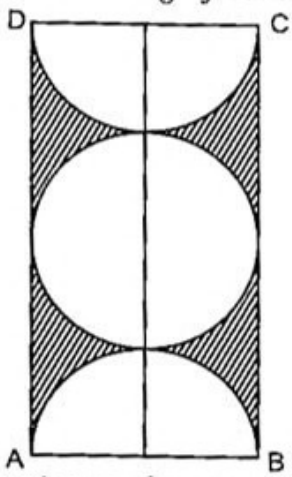
Since 46 has highest frequency

$\therefore \quad$ Mode = 46 **Ans.**

Question 4.

(a) *What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor ?* **[3]**

(b) *In the given figure ABCD is a rectangle. It consists of a circle and two semi circles each of which are of radius 5cm. Find the area of the shaded region. Give your answer correct to three significant figures.*** **[4]**



(c) *Solve the following inequation and represent the solution set on a number line.* **[3]**

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, x \in I$$

Answer 4.

(a) Let the required number be K.

Let $f(x) = 16x^3 - 8x^2 + 4x + 7 - K$

$\because$ $(2x + 1)$ is a factor of $f(x)$

$$\therefore \quad f\left(-\frac{1}{2}\right) = 0$$

$$\Rightarrow 16 \times \left(-\frac{1}{2}\right)^3 - 8 \times \left(-\frac{1}{2}\right)^2 + 4 \times \left(-\frac{1}{2}\right) + 7 - K = 0$$

$$\Rightarrow -16 \times \frac{1}{8} - 8 \times \frac{1}{4} - 4 \times \frac{1}{2} + 7 - K = 0$$

$$\Rightarrow -2 - 2 - 2 + 7 - K = 0$$

$$\Rightarrow -6 + 7 - K = 0$$

$$\Rightarrow 1 - K = 0$$

$$\Rightarrow K = 1$$

$\therefore$ The required number = 1 **Ans.**

(c) Given inequation is,

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, x \in I$$

** **Answer is not given due to change in the present syllabus.**

$$\Rightarrow \quad -\frac{17}{2} < \frac{-1-8x}{2} \leq \frac{15}{2}$$

$$\Rightarrow \quad -\frac{17}{2} \times 2 < \frac{-1-8x}{2} \times 2 \leq \frac{15}{2} \times 2$$

$$\Rightarrow \quad -17 < -1-8x \leq 15$$

$$\Rightarrow \quad -17+1 < -1+1-8x \leq 15+1$$

$$\Rightarrow \quad -16 < -8x \leq 16$$

$$\Rightarrow \quad \frac{-16}{-8} > \frac{-8x}{-8} \geq \frac{16}{-8}$$

$$\Rightarrow \quad 2 > x \geq -2$$

$$\Rightarrow \quad -2 \leq x < 2$$

$\therefore$ Solution set $= \{-2, -1, 0, 1\}$



Ans.

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *Given matrix* $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. *Find the matrix* X *if,* $X = B^2 - 4B$. **[4]**

Hence solve for a and b given $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

(b) *How much should a man invest in ₹ 50 shares selling at ₹ 60 to obtain an income of ₹ 450, if the rate of dividend declared is 10%. Also find his yield percent, to the nearest whole number.* **[3]**

(c) *Sixteen cards are labelled as a, b, c, ……m, n, o, p. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is :* **[3]**

(i) a vowel.

(ii) a consonant.

(iii) none of the letters of the word median.

Answer 5.

(a) Given, $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$\therefore$

$$X = B^2 - 4B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Ans.

Now, $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5a+0 \\ 0+5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow 5a = 5$ and $5b = 50$

$\Rightarrow a = 1$ and $b = 10$ **Ans.**

(b) Given, N.V. = ₹ 50, M.V. = ₹ 60, Total income = ₹ 450
Rate of dividend = 10%

$\therefore$ Dividend per share $= \frac{10}{100} \times ₹\ 50 = ₹\ 5$

$\therefore$ Number of shares $(n) = \frac{₹\ 450}{₹\ 5} = 90$

$\therefore$ Investment $=$ M.V. $\times\ n$
$= ₹\ 60 \times 90 = ₹\ 5400$ **Ans.**

$\therefore$ Yield percent $= \frac{₹\ 5}{₹\ 60} \times 100\%$
$= \frac{25}{3}\% = 8{\cdot}33\%$
$= 8\%$ (to the nearest whole no.) **Ans.**

(c) Here, sample space (S) = $\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p\}$

$\therefore$ n (S) $= 16$

(i) Vowels, V $= \{a, e, i, o\}$

$\therefore$ n (V) $= 4$

$\therefore$ P (a vowel) $= \frac{n\text{ (V)}}{n\text{ (S)}}$
$= \frac{4}{16} = \frac{1}{4}$ **Ans.**

(ii) Consonants, C = $\{b, c, d, f, g, h, j, k, l, m, n, p\}$

$\therefore$ n (C) $= 12$

$\therefore$ P (a consonant) $= \frac{n\text{ (C)}}{n\text{ (S)}}$
$= \frac{12}{16} = \frac{3}{4}$ **Ans.**

(iii) None of the letters of the word median (N) = $\{b, c, f, g, h, j, k, l, o, p\}$

$\therefore$ n (N) $= 10$

$\therefore$ P (N) $= \frac{n\text{ (N)}}{n\text{ (S)}}$
$= \frac{10}{16} = \frac{5}{8}$ **Ans.**

Question 6.

(a) *Using a ruler and a compass construct a triangle ABC in which AB = 7cm, ∠CAB = 60° and AC = 5cm. Construct the locus of :* **[4]**

(i) points equidistant from AB and AC.

(ii) points equidistant from BA and BC.

Hence construct a circle touching the three sides of the triangle internally.

(b) *A conical tent is to accommodate 77 persons. Each person must have $16m^3$ of air to breathe. Given the radius of the tent as 7m find the height of the tent and also its curved surface area.* **[3]**

(c) *If* $\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$, *use properties of proportion to find* **[3]**

(i) $m : n$

(ii) $\frac{m^2 + n^2}{m^2 - n^2}$

Answer 6.

(a) Given, AB = 7 cm, ∠ CAB = 60° , AC = 5 cm

Steps of construction :

(1) Construct triangle ABC with given measurements.

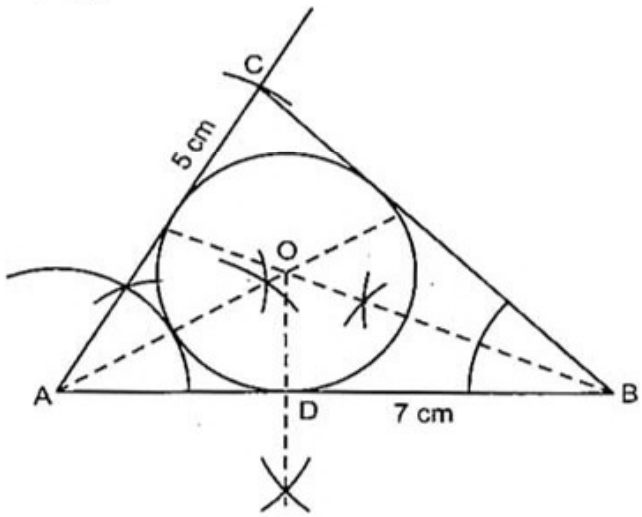
(2) Draw bisector of ∠BAC which is the locus of points equidistant from AB and AC.

(3) Draw bisector of ∠ABC which is the locus of points equidistant from AB and BC.

(4) Let the bisectors meet at O.

(5) Draw a perpendicular from O to AB intersecting AB at D.

(6) Taking O as centre and OD as radius, construct a circle touching the three sides of triangle internally.



(b) Given, number of persons = 77.

Volume of air required by each person = 16 m^3

∴ Total volume of air required for 77 persons = 77 × 16 m^3 = 1232 m^3.

Radius (r) = 7 m

Let the height of tent be h m.

We have $\text{Volume} = \frac{1}{3}\pi r^2 h$

$\Rightarrow 1232 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h$

$\Rightarrow h = \frac{1232 \times 3}{22 \times 7} = 24$ m

∴ Required height = 24 m

Now, slant height (l) $= \sqrt{h^2 + r^2}$

$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$

$= \sqrt{625} = 25$ m

$\therefore$ The curved surface area $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550$ m^2 **Ans.**

(c) (i) Given, $\frac{7m+2n}{7m-2n} = \frac{5}{3}$

Using componendo and dividendo,

$$\frac{(7m+2n)+(7m-2n)}{(7m+2n)-(7m-2n)} = \frac{5+3}{5-3}$$

$$\Rightarrow \quad \frac{7m+7m}{2n+2n} = \frac{8}{2}$$

$$\frac{14m}{4n} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{4}{1} \times \frac{2}{7}$$

$$\Rightarrow \quad m : n = 8 : 7$$ **Ans.**

(ii) $\frac{m}{n} = \frac{8}{7} \Rightarrow \frac{m^2}{n^2} = \frac{64}{49}$

Using componendo and dividendo,

$$\frac{m^2+n^2}{m^2-n^2} = \frac{64+49}{64-49} = \frac{113}{15}$$ **Ans.**

Question 7.

(a) *A page from a savings bank account passbook is given below :*** **[5]**

Data	*Particulars*	*Amount Withdrawn (₹)*	*Amount Deposited (₹)*	*Balance (₹)*
Jan. 7, 2016	*B/F*			*3000.00*
Jan. 10, 2016	*By Cheque*		*2600.00*	*5600.00*
Feb. 8, 2016	*To Self*	*1500.00*		*4100.00*
Apr. 6, 2016	*By Cheque*	*2100.00*		*2000.00*
May 4, 2016	*By Cash*		*6500.00*	*8500.00*
May 27, 2016	*By Cheque*		*1500.00*	*10000.00*

(i) Calculate the interest for the 6 months from January to June 2016, at 6% per annum.

(ii) If the account is closed on 1st July 2016, find the amount received by the account holder.

(b) *Use a graph paper for this question (Take 2 cms = 1 unit on both x and y axis)* **[5]**

(i) Plot the following points :

A(0, 4), B(2,3), C(1,1) and D(2,0)

(ii) Reflect points B, C, D on the y-axis and write down their coordinates. Name the images as B′, C′,D′ respectively.

** **Answer is not given due to change in the present syllabus.**

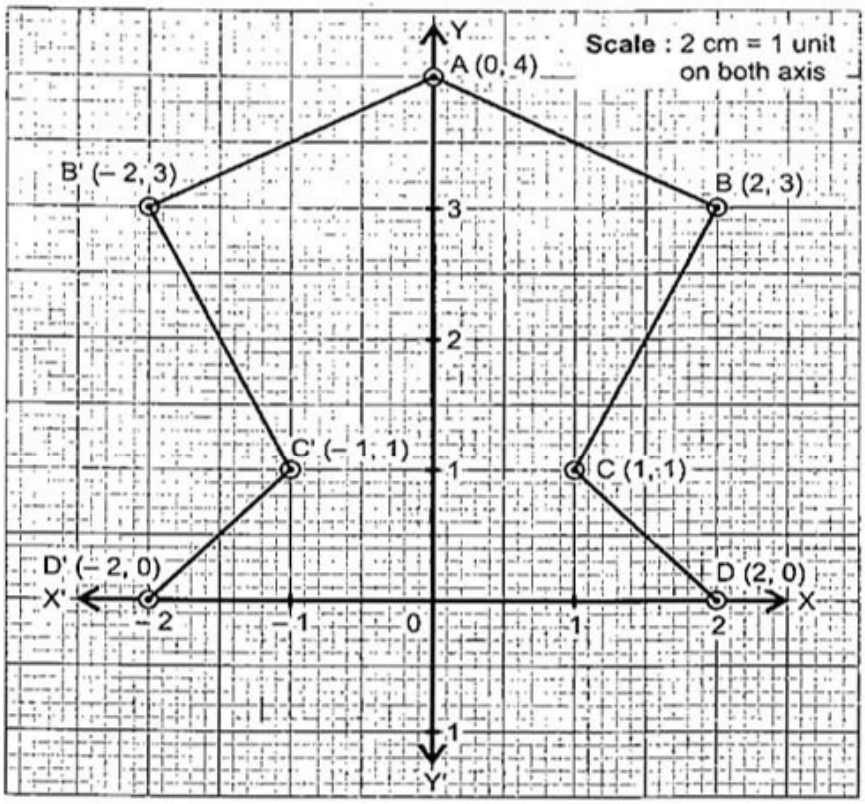
(iii) *Join the points A, B, C, D, D′, C′, B′ and A in order, so as to form a closed figure.*

*Write down equation of the line of symmetry of the figure formed.***

Answer 7.

(b) (i) On graph : A (0, 4), B (2, 3), C (1, 1), D (2, 0)

(ii) B′ = (– 2, 3), C′ = (– 1, 1), D′ = (– 2, 0)

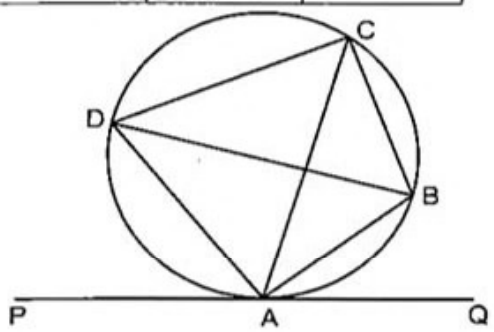


Question 8.

(a) *Calculate the mean of the following distribution using step deviation method.* **[4]**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of Students	10	9	25	30	16	10

(b) *In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of ∠CAQ and ∠PAC. If ∠BAQ = 30°, prove that :* **[3]**



(i) *BD is a diameter of the circle.*

(ii) *ABC is an isosceles triangle.*

** **Answer is not given due to change in the present syllabus.**

(c) *The printed price of an air conditioner is ₹ 45,000/-. The wholesaler allows a discount of 10% to the shopkeeper. The shopkeeper sells the article to the customer at a discount of 5% of the marked price. Sales tax (under VAT) is charged at the rate of 12% at every stage. Find :* **[3]**

(i) VAT paid by the shopkeeper to the government.

(ii) The total amount paid by the customer inclusive of tax.

Answer 8.

(a)

Marks	Mid values (x_i)	No. of students (f_i)	$d_i = x_i - A$	$t_i = \frac{d_i}{c}$	$f_i t_i$
0—10	5	10	– 20	– 2	– 20
10—20	15	9	– 10	– 1	– 9
20—30	25	25 = A	0	0	0
30—40	35	30	10	1	30
40—50	45	16	20	2	32
50—60	55	10	30	3	30
		$\Sigma f_i = 100$			$\Sigma f_i t_i = 63$

Let A = 25 and $c = 10$

$$\therefore \quad \text{Mean} = A + \frac{\Sigma f_i t_i}{\Sigma f_i} \times c$$

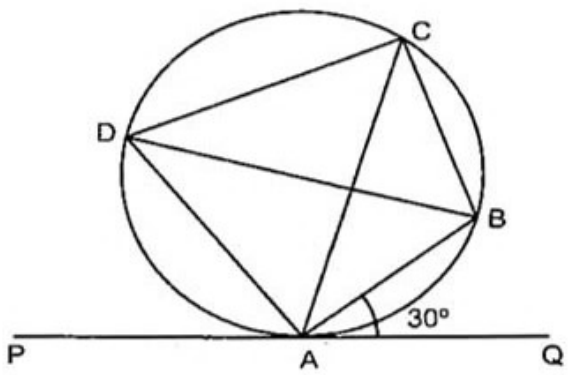
$$= 25 + \frac{63}{100} \times 10$$

$$= 25 + 6{\cdot}3$$

$$= 31{\cdot}3$$

Ans.

(b) Given, $\angle BAQ = 30°$, AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$



(i) $\angle BAC = 30°$

$= \angle BAQ$ (AB bisects $\angle CAQ$)

$\angle CAQ = \angle BAC + \angle BAQ$

$= 30° + 30° = 60°$

$\angle PAC = 180° - \angle CAQ$ (Linear pair)

$= 180° - 60° = 120°$

$\angle CAD = \frac{1}{2} \angle PAC$ (AD bisects $\angle PAC$)

$= \frac{1}{2} \times 120° = 60°$

$\angle BAD = \angle BAC + \angle CAD = 30° + 60° = 90°$

$\therefore$ BD is a diameter ($\angle BAD = 90°$ = angle in a semi-circle)

Hence Proved.

(ii) $\angle ADB = \angle BAQ$ (angle in an alternate segment)

$= 30°$

$\angle ACB = \angle ADB$ (angles in same segment are equal)

$\therefore \quad \angle BAC = \angle ACB = 30°$

$\therefore \quad AB = BC$ (sides opposite to equal angles are equal)

$\therefore$ ABC is an isosceles triangle **Hence Proved.**

(c) Given, Printed price = ₹ 45,000

For shopkeeper,

$$\text{Purchase price} = ₹\left(45000 - \frac{10}{100} \times 45{,}000\right)$$

$$= ₹\ 40{,}500$$

$$\therefore \quad \text{Input tax} = \frac{12}{100} \times 40{,}500 = ₹\ 4{,}860$$

$$\text{Sale price} = ₹\left(45{,}000 - \frac{5}{100} \times 45{,}000\right)$$

$$= ₹\ 42{,}750$$

$$\therefore \quad \text{Output tax} = \frac{12}{100} \times 42{,}750$$

$$= ₹\ 5{,}130$$

(i) VAT paid by shopkeeper = ₹ (5,130 – 4,860)

= ₹ 270 **Ans.**

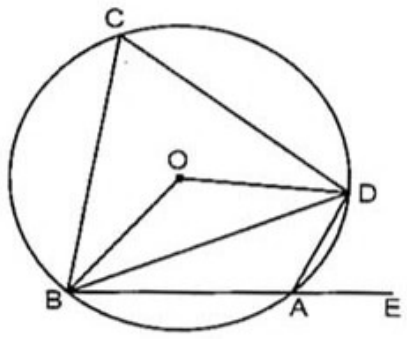
(ii) Total amount paid by the customer

= ₹ (42,750 + 5,130)

= ₹ 47,880 **Ans.**

Question 9.

(a) *In the figure given, O is the centre of the circle. ∠DAE = 70°. Find giving suitable reasons , the measure of :* **[4]**



(i) $\angle BCD$

(ii) $\angle BOD$

(iii) $\angle OBD$

(b) *A(– 1, 3), B(4, 2) and C(3, – 2) are the vertices of a triangle.* **[3]**

(i) Find the coordinates of the centroid G of the triangle.

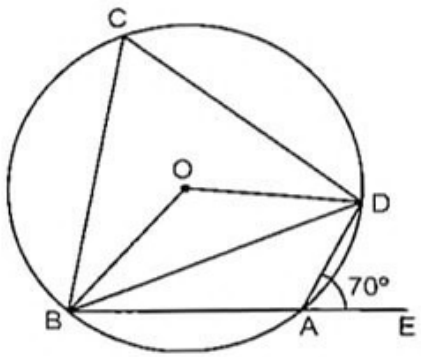
(ii) Find the equation of the line through G and parallel to AC.

(c) *Prove that* **[3]**

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

Answer 9.

(a) Given, $\angle DAE = 70°$



(i) $\angle BAD + \angle DAE = 180°$ (Linear pair)

$\Rightarrow \quad \angle BAD = 180° - 70° = 110°$

$\therefore \quad \angle BCD + \angle BAD = 180°$

(Sum of opposite angles of cyclic quadrilateral is 180°)

$\Rightarrow \quad \angle BCD = 180° - 110°$

$= 70°$ **Ans.**

(ii) $\angle BOD = 2 \angle BCD$

(Angle at centre is twice the angle at circumference)

$= 2 \times 70°$

$= 140°$ **Ans.**

(iii) $\angle OBD = \angle ODB$ ($OB = OD$ = radius)

$\therefore \quad \angle OBD + \angle ODB + \angle BOD = 180°$

(Sum of angles in a triangle is 180°)

$\Rightarrow \quad \angle OBD + \angle OBD + 140° = 180°$ ($\angle OBD = \angle ODB$)

$\Rightarrow \quad 2 \angle OBD = 180° - 140°$

$\Rightarrow \quad \angle OBD = \frac{40°}{2} = 20°$ **Ans.**

(b) Given, A $(-1, 3)$, B $(4, 2)$, C $(3, -2)$.

(i) Coordinates of centroid $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{-1 + 4 + 3}{3}, \frac{3 + 2 - 2}{3}\right)$

$= \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$ **Ans.**

(ii) Slope of $AC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{3 - (-1)} = \frac{-5}{4}$

$\therefore$ Slope of the required line $(m) = \frac{-5}{4}$

Let the equation of the line through G, be

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 1 = -\frac{5}{4}(x - 2)$$

$$\Rightarrow \quad 4y - 4 = -5x + 10$$

$\Rightarrow 5x + 4y - 14 = 0$, which is the required line. **Ans.**

(c)

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta\{1-2(1-\cos^2\theta)\}}{\cos\theta(2\cos^2\theta-1)}$$

$$= \frac{\sin\theta(1-2+2\cos^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \frac{\sin\theta(2\cos^2\theta-1)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{R.H.S.}$$ **Hence Proved.**

Question 10.

(a) *The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.* **[4]**

(b) *The daily wages of 80 workers in a project are given below.* **[6]**

Wages (in ₹)	*400–450*	*450–500*	*500–550*	*550–600*	*600–650*	*650–700*	*700–750*
No. of workers	2	6	12	18	24	13	5

Use a graph paper to draw an ogive for the above distribution. (Use a scale of 2 cm = ₹ 50 on x-axis and 2 cm = 10 workers on y-axis). Use your ogive to estimate :

(i) the median wage of the workers.

(ii) the lower quartile wage of workers.

(iii) the number of workers who earn more than ₹ 625 daily.

Answer 10.

(a) Let Vivek's present age be x years.

$\therefore$ His brother's age $= (47 - x)$ years.

According to question,

$$x(47-x) = 550$$

$$\Rightarrow 47x - x^2 = 550$$

$$\Rightarrow x^2 - 47x + 550 = 0$$

$$\Rightarrow x^2 - 25x - 22x + 550 = 0$$

$$\Rightarrow x(x-25) - 22(x-25) = 0$$

$$\Rightarrow (x-25)(x-22) = 0$$

$$\Rightarrow x - 25 = 0 \text{ or } x - 22 = 0$$

$$\Rightarrow x = 25 \text{ or } x = 22$$

When $x = 25, 47 - x = 47 - 25 = 22$

When $x = 22, 47 - x = 47 - 22 = 25$ (does not satisfy the given condition)

$\therefore$ Vivek's age $= x = 25$ years.

His younger brother's age $= 22$ years. **Ans.**

(b)

Wages (in ₹)	No. of Workers	Cumulative Frequency
400—450	2	2
450—500	6	8
500—550	12	20
550—600	18	38
600—650	24	62
650—700	13	75
700—750	5	80

$\therefore \quad n = 80$

(i) Median wage $= \frac{n}{2}$ th value.

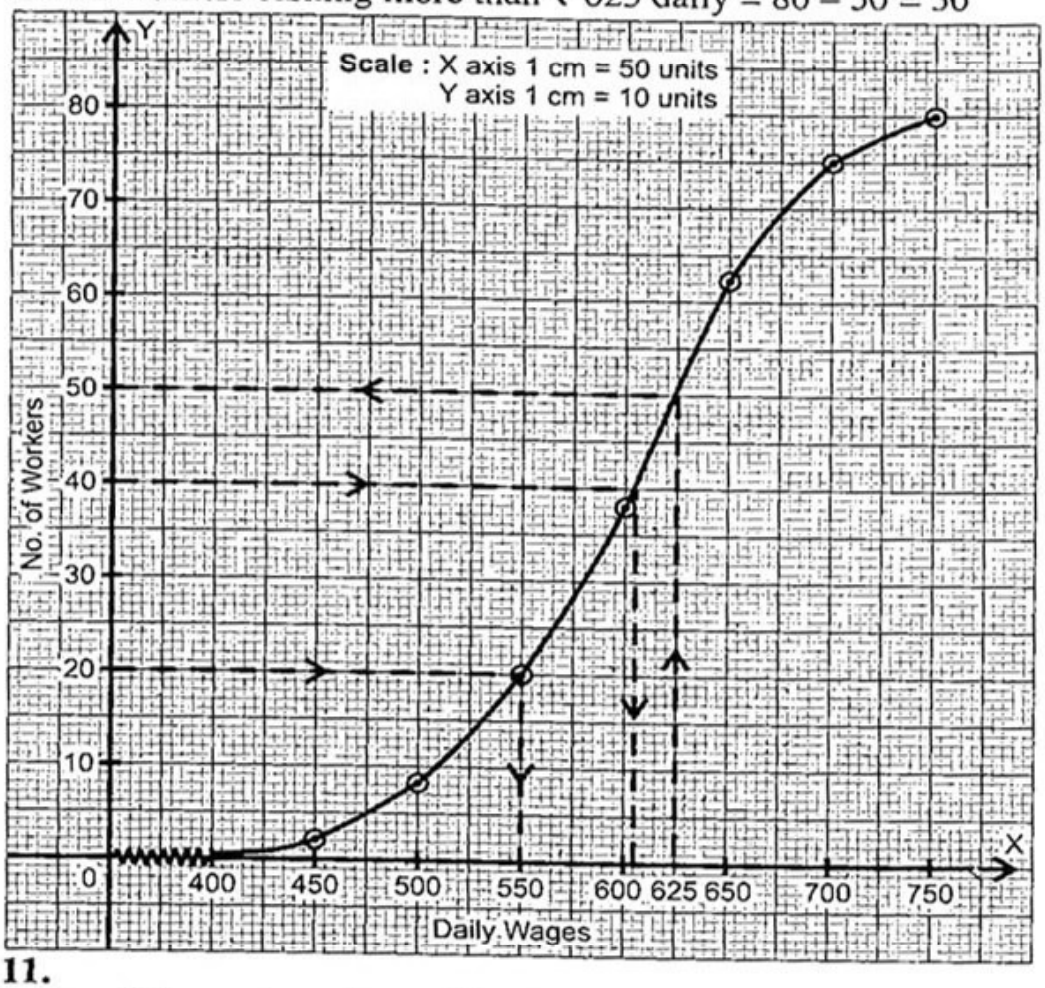
$= \frac{80}{2}$ th value

$=$ 40th value

$=$ ₹ 605 **Ans.**

(ii) Lower quartile $= \frac{n}{4}$ th value $=$ 20th value $=$ ₹ 550 **Ans.**

(iii) No. of workers earning more than ₹ 625 daily $= 80 - 50 = 30$ **Ans.**



Question 11.

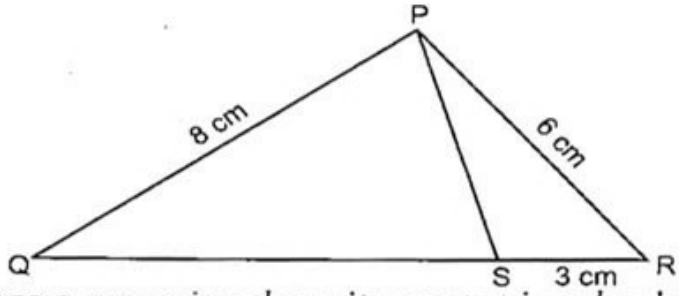
(a) *The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships. Give your answer correct to the nearest whole number.* **[4]**

(b) *PQR is a triangle. S is a point on the side QR of Δ PQR such that ∠ PSR = ∠ QPR. Given QP = 8 cm, PR = 6 cm and SR = 3 cm* **[3]**

(i) Prove ΔPQR ~ ΔSPR

(ii) Find the length of QR and PS

(iii) $\frac{\text{area of } \Delta PQR}{\text{area of } \Delta SPR}$



(c) *Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p.a. simple interest. If he gets ₹ 8325 as interest at the time of maturity, find :* **[3]**

(i) The monthly deposit *(ii) The maturity value.*

Answer 11.

(a) Let CD be the height of the Light house

$\therefore$ CD = 60 m.

Let AD = x m, BD = y m.

In Δ ACD,

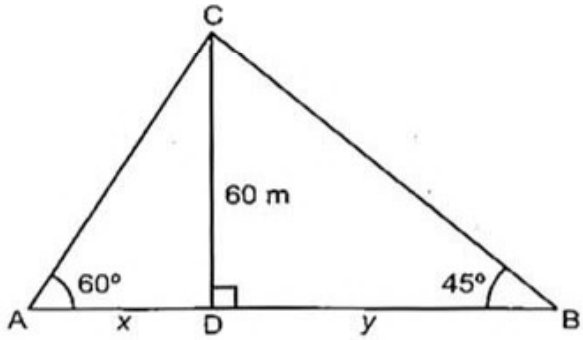
$$\tan 60° = \frac{CD}{AD}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x} \Rightarrow x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$$

$$= 20 \times 1{\cdot}732$$

$$= 34{\cdot}64 \text{ m}$$



In Δ BCD,

$$\tan 45° = \frac{CD}{BD}$$

$$\Rightarrow 1 = \frac{60}{y} \Rightarrow y = 60 \text{ m}$$

$\therefore$ Distance between two ships = $x + y$ = 34·64 + 60

= 94·64 m

= 95 m (correct to nearest whole number) **Ans.**

(b) Given, ∠PSR = ∠QPR,

QP = 8 cm, PR = 6 cm, SR = 3 cm.

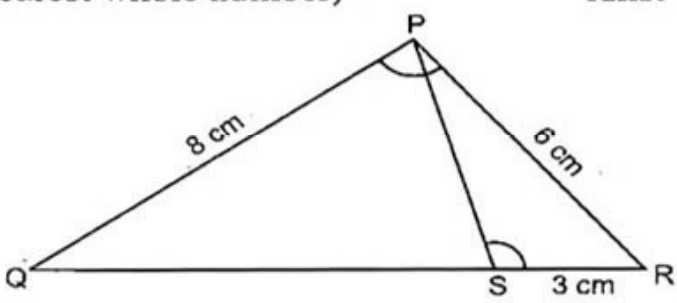
(i) In Δ PQR and Δ SPR

∠ PSR = ∠ QPR (Given)

∠ R = ∠ R (Common angle)

$\therefore$ Δ PQR ~ Δ SPR (AA axiom)

Hence Proved.



(ii) $\frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR}$ ($\because \Delta PQR \sim \Delta SPR$)

$\therefore \quad \frac{QR}{PR} = \frac{PR}{SR}$

$\Rightarrow \quad \frac{QR}{6} = \frac{6}{3}$

$\Rightarrow \quad QR = \frac{6 \times 6}{3} = 12$ cm **Ans.**

$\therefore \quad \frac{PQ}{SP} = \frac{PR}{SR}$

$\Rightarrow \quad \frac{8}{SP} = \frac{6}{3}$

$\Rightarrow \quad SP = \frac{8 \times 3}{6}$

$\Rightarrow \quad SP = 4$ cm **Ans.**

(iii) $\frac{\text{area of } \Delta PQR}{\text{area of } \Delta SPR} = \frac{PR^2}{SR^2}$

$= \frac{6^2}{3^2}$

$= \frac{36}{9} = \frac{4}{1} = 4 : 1$ **Ans.**

(c) No. of months $(n) = 3 \times 12 = 36$, R = 7·5%

Interest = ₹ 8325

(i) Let monthly deposit be ₹ x

$\therefore \quad$ P for 1 month $= x \times \frac{n(n+1)}{2}$

$= x \times \frac{36 \times 37}{2}$

$= 666x$

$\therefore \quad$ Interest $= \frac{666x \times 1 \times 7{\cdot}5}{12 \times 100}$

$= \frac{333x}{80}$

According to the question,

$\frac{333x}{80} = 8325$

$\Rightarrow \quad x = \frac{8{,}325 \times 80}{333}$

$= ₹\ 2{,}000$

$\therefore$ Monthly deposit = ₹ 2,000 **Ans.**

(ii) Total deposits = ₹ 2,000 × 36 = ₹ 72,000

$\therefore \quad$ Maturity value = ₹ (72,000 + 8,325)

= ₹ 80,325 **Ans.**

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